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A THREE-DIMENSIONAL HYDRODYNAMIC MODEL OF THE GULF OF GDAŃSK

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Abstract

A three-dimensional, baroclinic, hydrodynamic model was based on the coastal ocean circulation model (Blumberg and Mellor 1987) known as POM (Princeton Ocean Model), which was adapted to Baltic conditions. The way of horizontal advection calculation has been modified by using TVD filtration (Total Variation Diminishing) thus oscillations in frontal zones have been eliminated. To predict water exchange between the Gulf of Gdańsk and the open sea, the model covers the whole Baltic Sea together with the Danish Straits. The model has been verified based on three-year simulation of spreading of freshwater introduced by 49 biggest rivers. The influence of wind has been taken into account assuming the variable fields of shear stress every 6 hours. Thermic conditions have been affected by heat fluxes calculated from meteorological data. Calculated and measured salinity and temperature distributions are in relatively good accordance for the surface layer. Seasonal changes in vertical distributions of temperature as well as formation and disappearance of thermocline have been approximated properly.

INTRODUCTION

In the past years, an increased inflow of pollutants into the Gulf of Gdańsk has raised a great deal of interest in mathematical methods of modelling of the propagation of substances introduced from land (Jędrasik and Kowalewski

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1993, Ołdakowski *et al*. 1994, van der Vat 1994). An appropriately designed hydrodynamic model enables simulation of pollutant transport. The first three-dimensional model applied for such a purpose in the region of the Gulf of Gdańsk was TRISULA model (Robakiewicz and Karelse 1994).

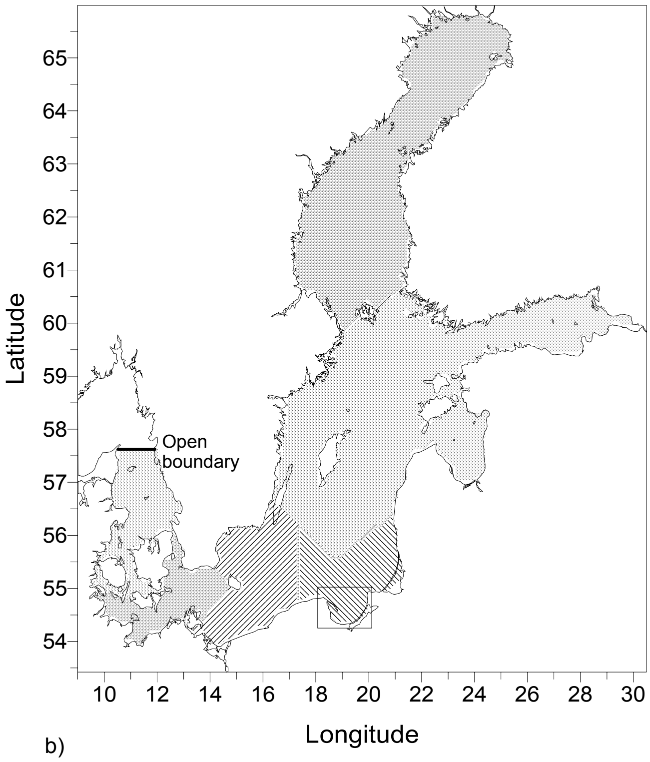
In this paper, a hydrodynamic model developed within the framework of the project: *A model of matter exchange and flow of energy in the Gulf of Gdańsk ecosystem*, realized at Institute of Oceanography, University of Gdańsk, is presented. The model has been designed to perform long-term (several years) simulations of propagation of different substances in the Gulf of Gdańsk. Since between the Gulf of Gdańsk and the Baltic Sea there is a great water exchange it was necessary to make this model applicable for the whole Baltic region.

Theoretical and numerical solutions of the model have been based on POM (Blumberg and Mellor 1987), with some necessary modifications allowing its application for the Baltic Sea. Similarly as in POM, to parameterize vertical mixing processes, the scheme of second order turbulence closure was used (Mellor and Yamada 1982).

To obtain a proper resolution and reliable data, a numerical grid of short spatial steps (1–2 km) should be used. Since the calculations carried out in the Baltic Sea region would then require great computational power, a model with local grid density have been applied; with longer step in the Baltic Sea (*ca.* 10 km) and a shorter one in the Gulf of Gdańsk (*ca.* 2 km). The regions modelled are presented in Fig. 1. To obtain a proper approximation of water exchange with the North Sea, the Baltic Sea comprises also the Danish Straits. The open boundary was situated between the Kattegat and Skagerrak where radiation boundary conditions were accepted for flows.

The model was tested in three-year simulation, from January 1994 till December 1996. Since the main task of the model was the best approximation of advective-diffusive mass and energy transport, the verification was based on comparison between calculated and measured water salinity and temperature at different stations in the Gulf of Gdańsk. Salinity is a very good parameter to test hydrodynamic transport because in marine environment it does not undergo any transformations. It can be assumed that salinity of all the rivers flowing into the investigated water region equals zero and the measurements are relatively accurate. The comparison of modelled temperature distributions with real thermal conditions determines proper prediction of spread of biogenic substances and other compounds which undergo transformations in sea water. Particular attention was drawn to seasonal changes in vertical distributions of temperature and salinity, formation and disappearance of thermocline, and the depth of halocline occurrence. Proper approximation of stratification and its changes, typical of the Gulf of Gdańsk, confirms the model's correctness and is necessary for realistic simulation of baroclinic circulation.

a)



b)

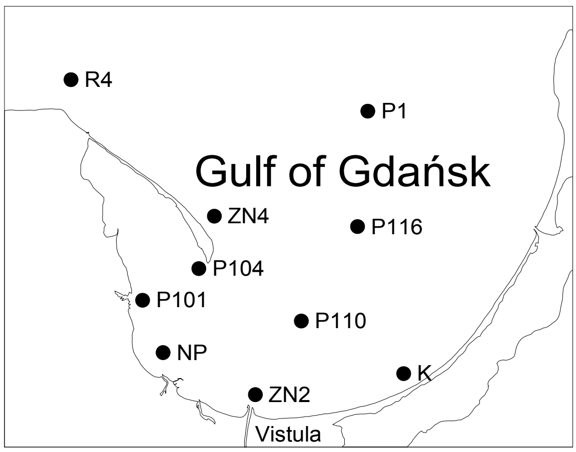


Fig. 1. The modelled regions: the Baltic Sea divided into the areas for which the climatic fields were averaged (a), the Gulf of Gdańsk with localization stations used for verification purposes (b)

THE GOVERNING EQUATIONS AND BOUNDARY CONDITIONS

The model has been based on the system of thermohydrodynamic equations in Boussinesq approximation, presented in Cartesian coordinate system with x, y and z increasing eastward, northward and upwards, respectively.
Motion equations have the following form

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}-fv=-\frac{1}{\rho_0}\frac{\partial p}{\partial x}+\frac{\partial}{\partial z}\left(K_{\mathrm{M}}\frac{\partial u}{\partial z}\right)+A_{\mathrm{M}}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right), \quad (1)$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}-fu=-\frac{1}{\rho_0}\frac{\partial p}{\partial y}+\frac{\partial}{\partial z}\left(K_{\mathrm{M}}\frac{\partial v}{\partial z}\right)+A_{\mathrm{M}}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right), \quad (2)$$

where:
u, v and w, components of velocity vector; f, Coriolis parameter; ρ and ρ_0, *in situ* and reference water densities; g, gravitational acceleration; p, pressure, K_{M} and A_{M}, the coefficients of vertical and horizontal momentum mixing.

The pressure at depth z can be calculated from

$$p(x,y,z,t) = p_{\mathrm{atm}} + g\rho_0\eta + g\int_z^0 \rho(x,y,z',t)dz', \quad (3)$$

where:
p_{atm}, atmospheric pressure; η, free surface elevation.

Continuity equation is given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0. \quad (4)$$

The conservation equations for temperature and salinity are

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\frac{\partial}{\partial z}\left(K_{\mathrm{H}}\frac{\partial T}{\partial z}\right)+A_{\mathrm{H}}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\theta_{\mathrm{T}}, \quad (5)$$

$$\frac{\partial S}{\partial t}+u\frac{\partial S}{\partial x}+v\frac{\partial S}{\partial y}+w\frac{\partial S}{\partial z}=\frac{\partial}{\partial z}\left(K_{\mathrm{H}}\frac{\partial S}{\partial z}\right)+A_{\mathrm{H}}\left(\frac{\partial^2 S}{\partial x^2}+\frac{\partial^2 S}{\partial y^2}\right), \quad (6)$$

where:

T, water temperature; S, salinity; K_H and A_H, coefficients of vertical and horizontal mass and heat exchange; θ_T, heat sources.

Values of density are computed from an equation of state

$$\rho = \rho(T, S, p), \tag{7}$$

in accordance with UNESCO standard (UNESCO 1983).

To solve the above system of equations, it is necessary to know the coefficients of turbulent momentum, mass and heat mixing. The coefficients of horizontal mixing can be determined from the Smagorinsky equation

$$A_M = A_C \Delta x \Delta y \sqrt{\left(\frac{\partial u}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2}\ , \tag{8}$$

where:

A_C, empirical coefficient; Δx and Δy, spatial spacing in x and y directions.

The vertical mixing coefficients (K_M and K_H) are obtained by applying a second order turbulence closure scheme (Mellor and Yamada 1982). The turbulence parameterization is based on the equations for the turbulence kinetic energy q^2 and the turbulence macroscale ℓ according to

$$\begin{aligned} &\frac{\partial q^2}{\partial t} + u\frac{\partial q^2}{\partial x} + v\frac{\partial q^2}{\partial y} + w\frac{\partial q^2}{\partial z} = \frac{\partial}{\partial z}\left(K_q \frac{\partial q^2}{\partial z}\right) + \\ &+2K_M\left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right] + \frac{2g}{\rho_0}K_H\frac{\partial \rho}{\partial z} - \frac{2q^3}{B_1\ell} + A_M\left(\frac{\partial^2 q^2}{\partial x^2} + \frac{\partial^2 q^2}{\partial y^2}\right), \end{aligned} \tag{9}$$

$$\begin{aligned} &\frac{\partial q^2\ell}{\partial t} + u\frac{\partial q^2\ell}{\partial x} + v\frac{\partial q^2\ell}{\partial y} + w\frac{\partial q^2\ell}{\partial z} = \frac{\partial}{\partial z}\left(K_q \frac{\partial q^2\ell}{\partial z}\right) + \\ &+\ell E_1 K_M\left[\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2\right] + \frac{\ell E_1 g}{\rho_0}K_H\frac{\partial \rho}{\partial z} - \frac{q^3}{B_1}\widetilde{W} + A_M\left(\frac{\partial^2 q^2\ell}{\partial x^2} + \frac{\partial^2 q^2\ell}{\partial y^2}\right), \end{aligned} \tag{10}$$

where a wall proximity function, $\widetilde{W}$ is defined as

$$\widetilde{W} \equiv 1 + E_2\left(\frac{\ell}{\kappa L}\right)^2, \tag{11}$$

$$\frac{1}{L} \equiv \frac{1}{\eta - z} + \frac{1}{H + z}, \tag{12}$$

where:

K_q, coefficient of vertical kinetic energy mixing; κ, the von Karman constant; H, sea depth; B_1, E_1 and E_2, constants determined experimentally.

The coefficients K_M, K_H and K_q can be expressed as

$$K_M \equiv \ell q S_M, \tag{13}$$

$$K_H \equiv \ell q S_H, \tag{14}$$

$$K_q \equiv \ell q S_q. \tag{15}$$

The stability functions, S_M, S_H and S_q, can be determined from the following system of equations

$$S_M[6A_1A_2G_M] + S_H[1 - 2A_2B_2G_H - 12A_1A_2G_H] = A_2, \tag{16}$$

$$S_M[1 + 6A_1^2G_M + 9A_1A_2G_H] - S_H[12A_1^2G_H + 9A_1A_2G_H] = A_1(1 - 3C_1), \tag{17}$$

$$S_q = 0.20, \tag{18}$$

$$G_M \equiv \frac{\ell^2}{q^2}\sqrt{\left(\frac{\partial u}{\partial z}\right)^2 + \left(\frac{\partial v}{\partial z}\right)^2}, \tag{19}$$

$$G_H \equiv \frac{\ell^2}{q^2}\frac{g}{\rho_0}\frac{\partial p}{\partial z}. \tag{20}$$

The constants A_1, A_2, B_1, B_2, C_1, E_1 and E_2 are of universal character and were obtained in laboratory measurements (Mellor and Yamada 1982):

$A_1 = 0.92$	$B_1 = 16.6$	$C_1 = 0.08$	$E_1 = 1.8$
$A_2 = 0.74$	$B_2 = 10.1$		$E_2 = 1.33$

The boundary conditions at the sea surface, $z = \eta(x, y)$, are

$$\rho_0 K_M \frac{\partial u}{\partial z} = \tau_{0x}, \qquad \rho_0 K_M \frac{\partial v}{\partial z} = \tau_{0y}, \tag{21}$$

$$\rho_0 K_H \frac{\partial T}{\partial z} = H_0, \qquad \rho_0 K_H \frac{\partial S}{\partial z} = 0, \tag{22}$$

$$q^2 = B_1^{2/3} U_{*0}^2, \qquad q^2 \ell = 0, \tag{23}$$

$$w = u\frac{\partial \eta}{\partial x} + v\frac{\partial \eta}{\partial y} + \frac{\partial \eta}{\partial t}. \tag{24}$$

The boundary conditions at the bottom, $z = -H(x, y)$, are

$$\rho_0 K_M \frac{\partial u}{\partial z} = \tau_{bx}, \qquad \rho_0 K_M \frac{\partial v}{\partial z} = \tau_{by}, \tag{25}$$

$$\tau_{bx} = \rho_0 C_D \sqrt{u_b^2 + v_b^2}\, u_b, \qquad \tau_{by} = \rho_0 C_D \sqrt{u_b^2 + v_b^2}\, v_b, \tag{26}$$

$$\rho_0 K_H \frac{\partial T}{\partial z} = 0, \qquad \rho_0 K_H \frac{\partial S}{\partial z} = 0, \tag{27}$$

$$q^2 = B_1^{2/3} U_{*b}^2, \qquad q^2 \ell = 0, \tag{28}$$

$$w_b = -u_b \frac{\partial H}{\partial x} - v_b \frac{\partial H}{\partial y}, \tag{29}$$

$$C_D = \max\left[\left(\frac{\kappa}{\ln(\Delta z_b / z_0)}\right)^2, 0.0025\right], \tag{30}$$

where:

τ_{ox} and τ_{oy}, components of surface wind stress vector; τ_{bx} and τ_{by} components of bottom frictional stress; U_{*0} and U_{*b}, friction velocity at the surface and bottom; H_0, heat flux across the sea surface; C_D, drag coefficient; z_0, roughness parameter; Δz_b, half of bottom layer thickness; u_b, v_b and w_b, velocity components at the bottom layer.

NUMERICAL SOLUTION

The equations and boundary conditions were transformed from Cartesian coordinate system (x, y, z, t) to sigma system (x^*, y^*, σ, t^*) defined as

$$x^* = x, \quad x^* = y, \quad \sigma = \frac{z - \eta}{D}, \quad t^* = t, \tag{31}$$

where:
$D = H + \eta$.

The form of transformed equations (1), (2), (4), (5), (6), (9) and (10) can be found in Blumberg and Mellor (1987). In the new coordinate system, the vertical velocity, ω, is defined as normal velocity to the sigma surface and is linked to the vertical velocity, *w*, as follows

$$\omega = w + u\left(\sigma\frac{\partial D}{\partial x} + \frac{\partial \eta}{\partial x}\right) + v\left(\sigma\frac{\partial D}{\partial y} + \frac{\partial \eta}{\partial y}\right) + \sigma\frac{\partial D}{\partial t} + \frac{\partial \eta}{\partial t} . \tag{32}$$

In the sigma system, the boundary conditions (24) and (29) are
- at the surface ($z = \eta$, $\sigma = 0$) – $\omega = 0$;
- at the bottom ($z = –$H, $\sigma = –1$) – $\omega = 0$.

To increase a time step and thus decrease the amount of necessary computer calculations, a mode splitting technique was applied. This technique enables separate calculation of free surface elevation by two-dimensional model (external mode) and the other state variables by three-dimensional model (internal mode). In external and internal modes the, Courant-Friedrichs-Levy numerical stability conditions are linked to the velocity of propagation of surface and internal wave, respectively. Since the velocity of internal wave propagation is much slower than that of surface wave propagation, the time step in the external mode is repeatedly shorter than in the internal mode.

In the external mode, the momentum and continuity equations are obtained by integrating the adequate internal mode equations from $\sigma = –1$ to $\sigma = 0$ (Blumberg and Mellor 1987). As a result, the two-dimensional model for the vertically integrated velocities

$$U = \int_{-1}^{0} u d\sigma \text{ and } V = \int_{-1}^{0} v d\sigma \tag{33}$$

is obtained. The free surface elevation can be calculated from the continuity equation

$$\frac{\partial \eta}{\partial t} + \frac{\partial UD}{\partial x} + \frac{\partial VD}{\partial y} = 0 . \tag{34}$$

The bottom friction, advective and baroclinic terms, calculated in the internal mode are inserted into the external mode, whereas the free surface elevation, calculated in the external mode is used in the internal mode. Both models are solved simultaneously and the exchange of information takes place at every time step (internal mode). Since external and internal velocities are characterized by different truncation errors, an increasing difference between the computed values appears, as the time progresses in the model. To prevent such a situation, the horizontal velocities are corrected at every time step in such a way so as to equalize their mean values with the external velocity.

To solve the model presented above, an identical numerical scheme as in POM, described in detail by Blumberg and Mellor (1987) and Mellor (1996), was used. The only change was a different scheme for advection. In sigma system, the equation of mass or heat conservation can be written as

$$\frac{D^{n+1}C^{n+1} - D^{n-1}C^{n-1}}{2\Delta t} + \mathrm{Adv}(C^n) = \frac{1}{D^{n+1}}\frac{\partial}{\partial\sigma}\left(K_\mathrm{H}\frac{\partial C^{n+1}}{\partial\sigma}\right) + \mathrm{Dif}(C^{n-1}) + \theta_\mathrm{C}, \quad (35)$$

where:

C, state variables T or S; θ_C, source function for C variable; Adv(C), advection terms; Dif(C), diffusion terms; n-1, n and n+1, time levels.

Fig. 2 illustrates the computational grid, called in the literature Arakawa "C" – grid. The advective term can be written as the balance of C variable transport

$$\mathrm{Adv}(C^n_{i,j,k}) = \frac{X^n_{i-1,j,k} - X^n_{i,j,k}}{\Delta x} + \frac{Y^n_{i,j-1,k} Y^n_{i,j,k}}{\Delta y} + \frac{Z^n_{i,j,k-1} - Z^n_{i,j,k}}{\Delta\sigma_k}, \quad (36)$$

where:

X, Y and Z denote C transport across appropriate walls of selected control volume

$$X^n_{i,j,k} = \frac{D^n_{i,j} + D^n_{i+1,j}}{2} u^n_{i,j,k} Xcsv^n_{i,j,k}, \quad (37)$$

$$Y^n_{i,j,k} = \frac{D^n_{i,j} + D^n_{i,j+1}}{2} v^n_{i,j,k} Ycsv^n_{i,j,k}, \quad (38)$$

$$Z^n_{i,j,k} = \omega^n_{i,j,k} Zcsv^n_{i,j,k}, \quad (39)$$

where:

Xcsv, Ycsv and Zcsv, control surface values; $\Delta\sigma_k$, k layer thickness in sigma coordinates.

In POM, a central difference scheme was used, *i.e.* the values at control surface were the means of the values at adjacent grid points:

$$Xcsv_{i,j,k}^{n} = \frac{C_{i,j,k}^{n} + C_{i+1,j,k}^{n}}{2} . \tag{40}$$

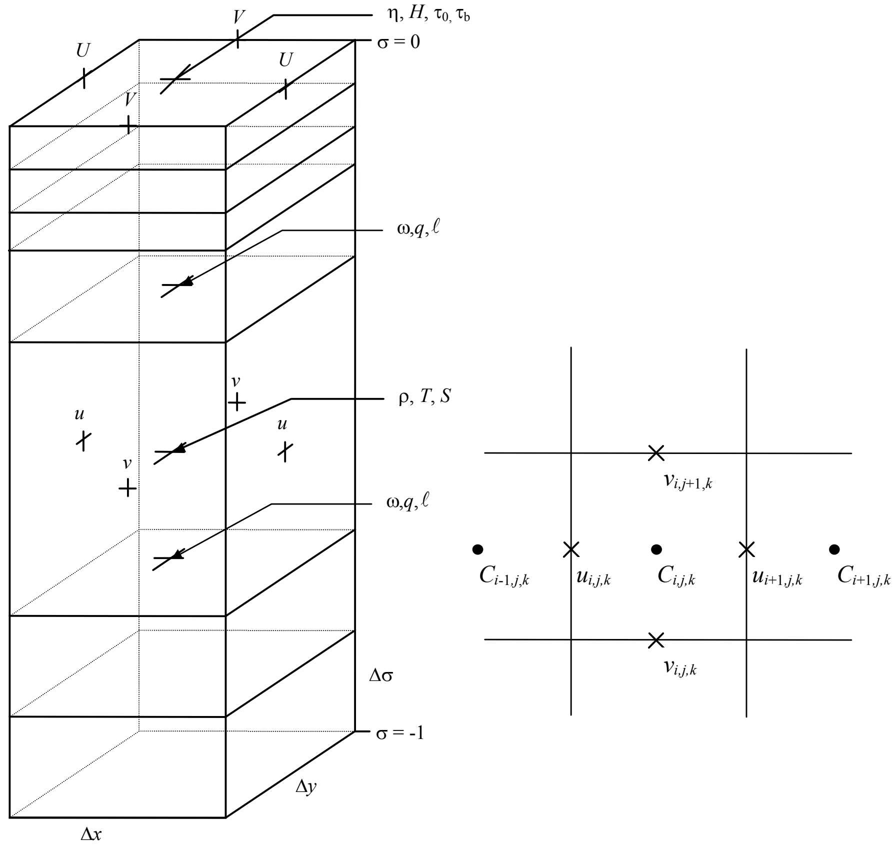


Fig. 2. The locations of variables on the numerical grid

The initial tests performed for the Baltic Sea show that in the case of advection the central difference scheme generates oscillations and local extremes with unreal values (*e.g.* negative salinity) in the regions of steep salinity gradients. This could be due to the fact that the scheme is not monotonic (Vested *et al.* 1996) and it does not hold for the water regions where hydrological fronts occur. The attempts of applying a simple monotonic scheme - upstream also do not give satisfactory results because of the great numerical dispersion caused by truncation errors. In consequence, strongly smoothed solutions were obtained. It appears that the application of ULTIMATE (Universal Limiter for Transient Interpolation Modelling of the Advective Transport Equations) scheme based on TVD (Total Variation Diminishing) filter (Leonard 1991) is a good approach. This scheme ensures monotonicity of any one-dimensional, explicit advection numerical scheme. In this case, it was combined with central difference scheme, separately for every direction. The algorithm of transport calculations along x could be formulated as follows

$Ccsv = Xcsv^{n}_{i,j,k}$ – compute the control surface value using the central differences scheme

$del = C^{n-1}_{i+1,j,k} - C^{n-1}_{i-1,j,k}$

$adel = \text{abs}(del)$

$acurv = \text{abs}\,(C^{n-1}_{i+1,j,k} - C^{n-1}_{i,j,k} + C^{n-1}_{i-1,j,k})$

if $acurv \geq adel$ then $Ccsv = C^{n-1}_{i,j,k}$

else

$Cref = C^{n-1}_{i-1,j,k} + (C^{n-1}_{i,j,k} + C^{n-1}_{i-1,j,k}) / CN_x$

if $del > 0$ then

if $Ccsv < C^{n-1}_{i,j,k}$ then $Ccsv = C^{n-1}_{i,j,k}$

if $Ccsv > Cref$ then $Ccsv = Cref$

endif

if $del < 0$ then

if $Ccsv > C^{n-1}_{i,j,k}$ then $Ccsv = C^{n-1}_{i,j,k}$

if $Ccsv < Cref$ then $Ccsv = Cref$

endif

endif

$Xcsv^{n}_{i,j,k} = Ccsv$

where:

$CN_x = \dfrac{u\Delta t}{\Delta x}$ is Courant number.

The above scheme is valid for positive values of u, but it should be modified for negative values. The same algorithm applies to the other axes.

It should be noted that three-dimensional version of ULTIMATE algorithm does not ensure absolute monotonicity because of separate filtration in different directions. However, the diffusive term in equation (35) acts as a stabilizer and in actual tests the oscillations are not noted. The impact of the filter modifies the values on control surface value in such situations which could create a new extreme. In typical marine conditions, when the gradients of C parameter are not steep, the filter leaves the value calculated by central difference scheme. In the case of steep C gradients or considerable flux velocities (when Courant number approaches value 1), the upstream scheme should be applied. Except for the above extreme cases, ULTIMATE algorithm enables one to calculate some intermediate values.

In the accepted numerical scheme, the time step is limited by appropriate conditions of numerical stability (Blumberg and Mellor 1987). The most important is Courant-Friedrichs-Levy condition:

$$\Delta t < \frac{1}{C_\mathrm{t}}\left(\frac{1}{\Delta x^2}+\frac{1}{\Delta y^2}\right)^{-\frac{1}{2}}, \tag{41}$$

where:

$C_\mathrm{t} = 2\sqrt{gH} + U_{\max}$ for the external mode, $U_{\max}$, maximum average velocity expected; $C_\mathrm{t} = 2C_\mathrm{i} + u_{\max}$ for the internal mode, C_i, maximum internal wave speed, $u_{\max}$, maximum advective speed.

Rotational condition

$$\Delta t < \frac{1}{2\Omega \sin\theta}, \tag{42}$$

where:

Ω, angular velocity of the earth; θ, latitude

and a condition connected with horizontal diffusion

$$\Delta t < \frac{1}{4A_\mathrm{H}}\left(\frac{1}{\Delta x^2}+\frac{1}{\Delta y^2}\right)^{-1} \tag{43}$$

usually do not limit the time step. Vertical diffusion also does not limit the time step because, as in POM, an implicit calculation scheme in vertical direction is applied (Mellor 1996).

APPLICATION OF THE MODEL TO THE BALTIC SEA AND THE GULF OF GDAŃSK

Because of the open character of the Gulf of Gdańsk, in hydrodynamic transport modelling the water exchange with the Baltic Sea has to be taken into account. The choice of appropriate boundary conditions at the border between the Gulf of Gdańsk and the open sea is very difficult because it requires the knowledge of temporal and spatial distributions of temperature, salinity, velocity and free surface elevation. In practice, such data can be obtained only from hydrodynamic models. Possible simplifications (*e.g.* the radiation boundary conditions) can result in considerable errors. For that reason, the model comprises two grids with different spatial steps (Tab. 1) for the two water regions analysed: *ca.* 5 nautical miles for the Baltic Sea and 1 nautical mile for the Gulf of Gdańsk. In both regions, calculations are performed simultaneously and the exchange of common boundary information takes place at every time step of the Baltic model. All model variables (T, S, q, ℓ, u, v, η, U, V) calculated at the boundary of one region serve as boundary conditions for the other. The algorithm realising connection ensures mass and energy conservation.

Table 1

Temporal and spatial steps in the modelled regions

Step	The Baltic Sea	The Gulf of Gdańsk
Δt (external mode)	1 min	20 s
Δt (internal mode)	20 min	4 min
Δx	10' longitude (7.8 - 10.6 km)	2' longitude (*ca.* 2.2 km)
Δy	5' latitude (9.3 km)	1' latitude (1.8 km)

At the open boundary between the Kattegat and Skagerrak, the radiation boundary condition for vertically averaged velocities was applied

$$V = \eta\sqrt{gH}\ . \tag{44}$$

Positive and negative values of the free surface elevation, η, cause water outflow from the Baltic Sea and water inflow from the Skagerrak, respectively. It was assumed that salinity distribution is constant during the year. At the open boundary, temperature distribution was the same as the one found for the nearest grid points.

The application of sigma transformation causes some errors in horizontal density gradient calculations (Haney 1991) resulting in the mistakes in calculated flows. To minimize such errors, a technique of subtraction of the area-averaged density before density gradient evaluation (Gary 1973, Mellor *et al.* 1994) was applied. Since in different parts of the Baltic Sea the vertical temperature and salinity distributions, and in consequence also density, differ significantly, the Baltic region was divided into some smaller areas where the averaging was performed separately (Fig. 1).

The model consists of 10 irregular levels. Their distribution is presented in Tab. 2. To improve transformation of surface and bottom wall proximity layers, their thickness, Δσ, was smaller than that of the central layers.

Table 2

The vertical σ coordinate distribution

Level	σ	$\Delta\sigma$
	0	
1		0.0714
	–0.0714	
2		0.0714
	–0.1429	
3		0.1429
	–0.2857	
4		0.1429
	–0.4286	
5		0.1429
	–0.5714	
6		0.1429
	–0.7143	
7		0.1429
	–0.8571	
8		0.0714
	–0.9286	
9		0.0357
	–0.9643	
10		0.0357
	–1.0000	

The initial temperature and salinity distributions were interpolated by DAS (Data Assimilation System) system (Sokolov *et al.* 1997) based on the data obtained during observations carried out in January 1994. The inflows from 49 rivers, together with 10 rivers flowing into the Gulf of Gdańsk, were considered. In the case of the Vistula and Reda, the two biggest rivers flowing into the Gulf of Gdańsk, the boundary conditions were assumed based on daily inflows and water temperature measurements. In the case of other rivers, the daily inflows and temperatures were calculated from trigonometric series describing seasonal variability in river outflows and determined from many years data (Cyberski 1997). The salinity was accepted as zero.

The fields of wind stress on the sea surface, averaged at 6-hour intervals, were taken from ECMWF (European Centre for Medium-Range Weather Forecasts) model. The heat flux across the water surface was calculated from the heat balance

$$H_0 = H_S + H_L + H_E + H_T \, , \tag{45}$$

where:

H_S, flux of short-wave solar radiation reaching the sea surface; H_L, flux of long-wave radiation from the sea surface; H_E, energy flux connected with water evaporation or vapour condensation; H_T, flux of sensibility heat.

The formulas for H_s calculation can be found in Krężel (1997), and for other components of the balance in Jędrasik (1997). The heat flux was calculated at every time step (internal mode). The fields of meteorological parameters (atmospheric pressure, temperature, air humidity) used in H_0 calculation were obtained by interpolation of daily mean data from *ca*. 100 coastal meteorological stations. The cloudiness fields above the Baltic Sea, calculated for 4 time points a day, were taken from ECMWF model. It was assumed that the flux of water evaporated from the water surface is equal to the flux of water from atmospheric precipitation.

VERIFICATION OF THE MODEL

To test the model precision, three-year simulation (January 1994 – December 1996) of hydrodynamic conditions in the Gulf of Gdańsk was performed. In calculations, no procedures of data assimilation were used. The fields of individual parameters were affected only by boundary conditions (river inflows and heat flux across the surface). Revision of the model was based on two parameters: salinity and temperature. Salinity is a good index of river water propagation because in marine environment it does not undergo any changes and its measurements are relatively accurate. Temperature, on the other hand, has a significant influence on the transformations of biogenic substance. The computed salinity and temperature data were compared with measured values for 10 stations in the Gulf of Gdańsk (Fig. 1).

Figs 3 and 4 present the comparison of modelled and observed changes in salinity and temperature for he mentioned three-year simulation.

Station P116, situated in the central part of the Gulf of Gdańsk, and three depth levels: surface (0–5 m), middle (40–50 m) and bottom (80–90 m), were chosen. At every depth level, the calculated temperatures are consistent with the measured ones. However, in the deepest layer, the seasonal temperature changes show a certain shift in phase (advance). On the other hand, the computed salinity is in agreement with the observations only in the surface and middle layers. In both cases, a slight tendency to gradual underestimating of calculated salinity values, especially for the 2nd and 3rd year of simulation, is noticeable. The computed salinity values are markedly underestimated in the bottom layer. Simplification of boundary conditions at the open Baltic boundary may lead to significant inaccuracies in determined exchange of water masses between the Baltic and the North Sea and that predicted by the model. Underestimated inflows of highly saline waters from the North Sea can result in lower salinity

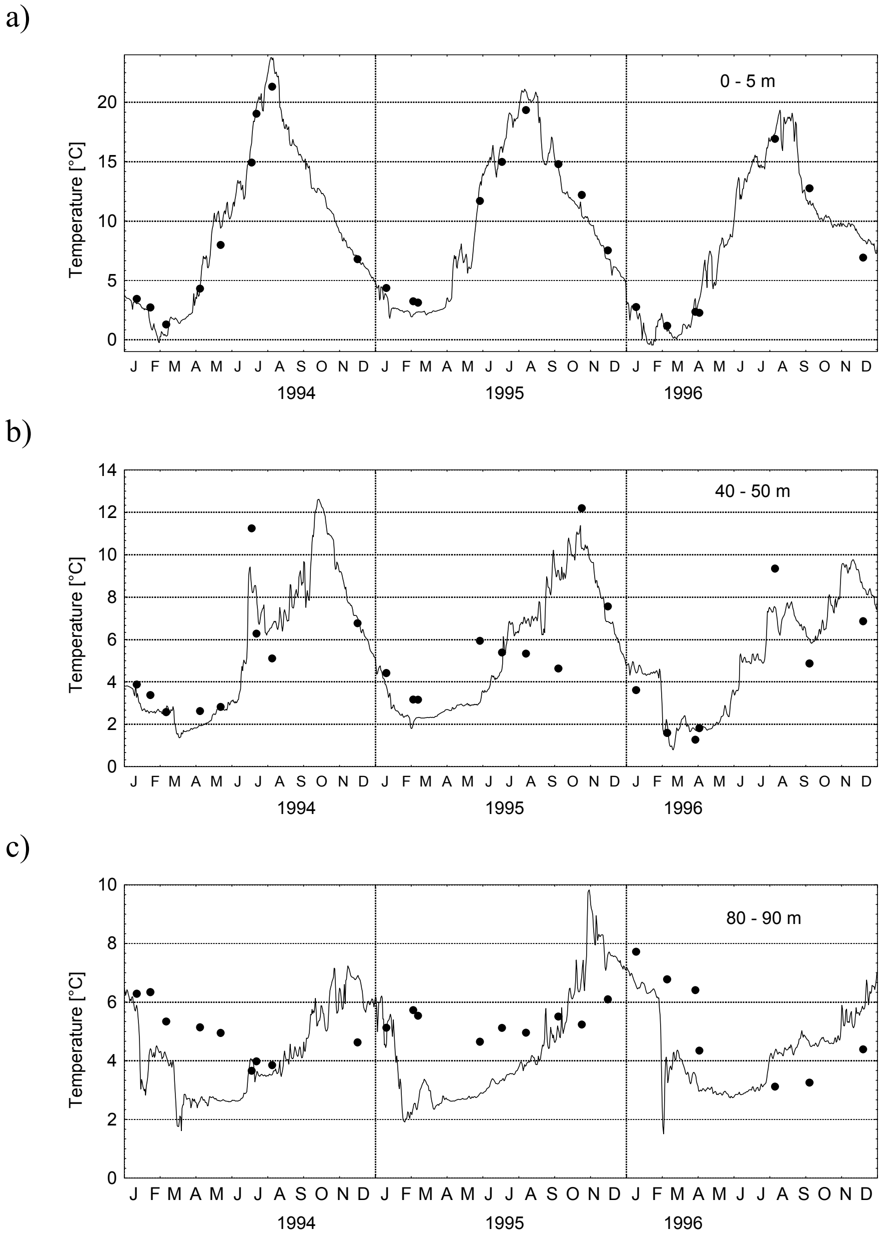
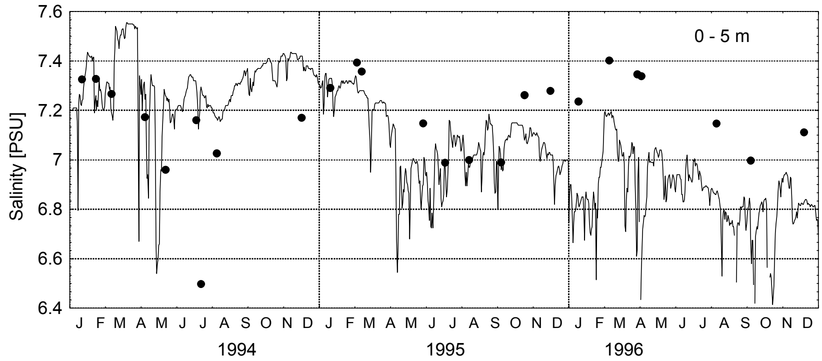


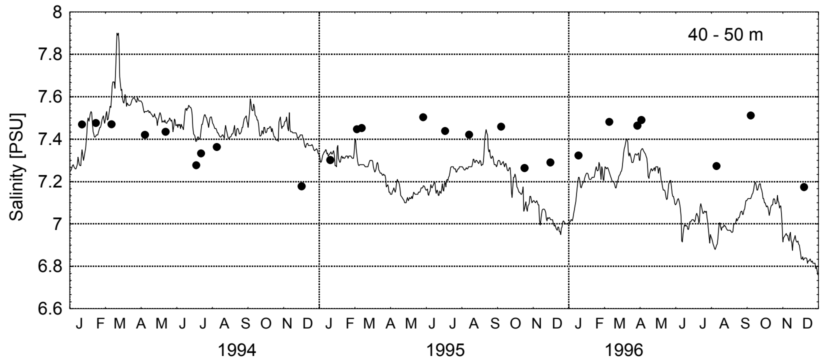
Fig. 3. The changes in modelled (line) and observed (points) temperature values during three-year simulation at station P116, at a depth of: 0–5 m (a), 40–50 m (b), 80–90 m (c)

values calculated by the model in the bottom layer and tendency to gradual decreasing of salinity in the upper layers.

a)



b)



c)

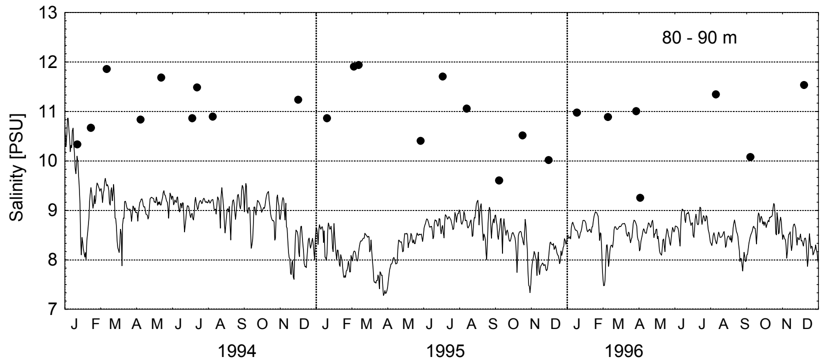


Fig. 4. The changes in modelled (line) and observed (points) salinity values during three-year simulation at station P116, at a depth of: 0–5 m (a), 40–50 m (b), (c), 80–90 m (c)

The comparison of the calculated data with the measurement results at all stations and for different depths in the Gulf of Gdańsk is illustrated in Tab. 3. The standard deviations and correlation coefficients between observed and modelled data were calculated. The highest conformity, *i.e.* low standard deviations and high correlation coefficients, were noted in the case of temperature values of the surface layer. The correlation coefficient decreases significantly with the depth, and standard deviation remains more or less at the same level. In the case of salinity, three levels characterized by different conformity of calculated and measured values could be distinguished. The highest conformity shows the surface layer with high correlation coefficients and low standard deviations. Low values of standard deviations and correlation coefficients in the middle layer (20–60 m) result from little variability of salinity at those depths. The lowest conformity was noted in the layer below 60 m. High values of standard deviations result from systematic underestimation of modelled salinity data in halocline layer and below it.

Table 3

Statistical characteristics of modelled and observed temperature and salinity data at different depths

Level [m]	Temperature		Salinity		Number of data
	Correlation coefficient	Standard deviation	Correlation coefficient	Standard deviation	
0–5	0.98	1.2	0.80	0.42	266
5–10	0.98	1.2	0.58	0.38	234
10–20	0.94	1.6	0.50	0.30	208
20–30	0.93	1.5	0.26	0.22	106
30–40	0.84	1.9	0.07	0.24	100
40–50	0.79	1.8	0.08	0.28	131
50–60	0.70	1.8	0.01	0.43	131
60–70	0.61	1.9	0.10	0.86	52
70–80	0.44	1.7	0.34	0.82	56
80–90	0.30	1.5	0.29	0.85	54
>90	0.24	1.4	0.27	0.95	31

The comparison of calculated and measured results in 0–60 m layer at different measuring stations in the Gulf of Gdańsk is presented in Tab. 4. Temperature values show very high correlation coefficients at every station. However, the standard deviations are slightly higher in the northern, open part of the Gulf (stations R4, ZN4 and P1). It indicates that in this region, the conformity of the model and observations is somewhat poorer. In the western and southern parts of the Gulf, the lowest standard deviations were obtained (stations P101, NP, K and ZN2). The results of salinity verification were similar but in this case, the both estimators must be analyzed simultaneously because the results are less unequivocal. The highest correlation coefficient (0.77) and highest standard deviations were noted at station ZN2, situated near the Vistula estuary. It is connected with great salinity changes in this region. The lowest correlation coefficients and low standard deviations were found at stations situated near the open boundary, between the Gulf of Gdańsk and the Baltic Sea.

Table 4

Statistical characteristics of modelled and observed temperature and salinity data at different measuring stations

Station	Temperature		Salinity		Number of data
	Correlation coefficient	Standard deviation	Correlation coefficient	Standard deviation	
R4	0.91	2.19	0.34	0.28	71
P1	0.96	1.36	0.43	0.21	155
ZN4	0.93	1.70	0.43	0.35	161
P116	0.95	1.52	0.39	0.22	150
P104	0.91	1.87	0.48	0.34	161
P101	0.98	1.08	0.64	0.27	55
P110	0.96	1.42	0.46	0.28	203
NP	0.99	1.16	0.52	0.35	60
K	0.97	1.26	0.51	0.53	66
ZN2	0.97	1.51	0.77	0.61	94

The vertical distributions of calculated and measured temperature and salinity values at station P116 in winter and summer are presented in Figs 5 and 6.

In individual years, a great conformity of temperature simulation with measured data was noted. The model correctly approximates the thermocline location (with the exception of August 96). In winter, a systematic underestimation of values (*ca*. 1 °C) could be sometimes observed in the whole profile. Except for the above described salinity underestimation at greater depths, the vertical profiles of calculated salinity show conformity. The depth of halocline deposition is calculated correctly.

CONCLUSIONS

This paper presents a detailed description of a three-dimensional, hydrodynamic model of the Gulf of Gdańsk. The model has been based on the coastal ocean circulation model POM. Numerical scheme of horizontal advection has been modified by using TVD filter, thus oscillations and unreal values in frontal zones have been eliminated. To ensure the boundary conditions at the open Gulf boundary, the model included the Baltic Sea region. The model comprises the Danish Straits region which enables the water exchange with the North Sea. It was necessary to divide the Baltic Sea into some areas where the climatic fields of temperature, salinity and density have been averaged separately.

The model verification consists in comparison of three-year simulation results with the measured data observed at 10 measuring stations. In the surface

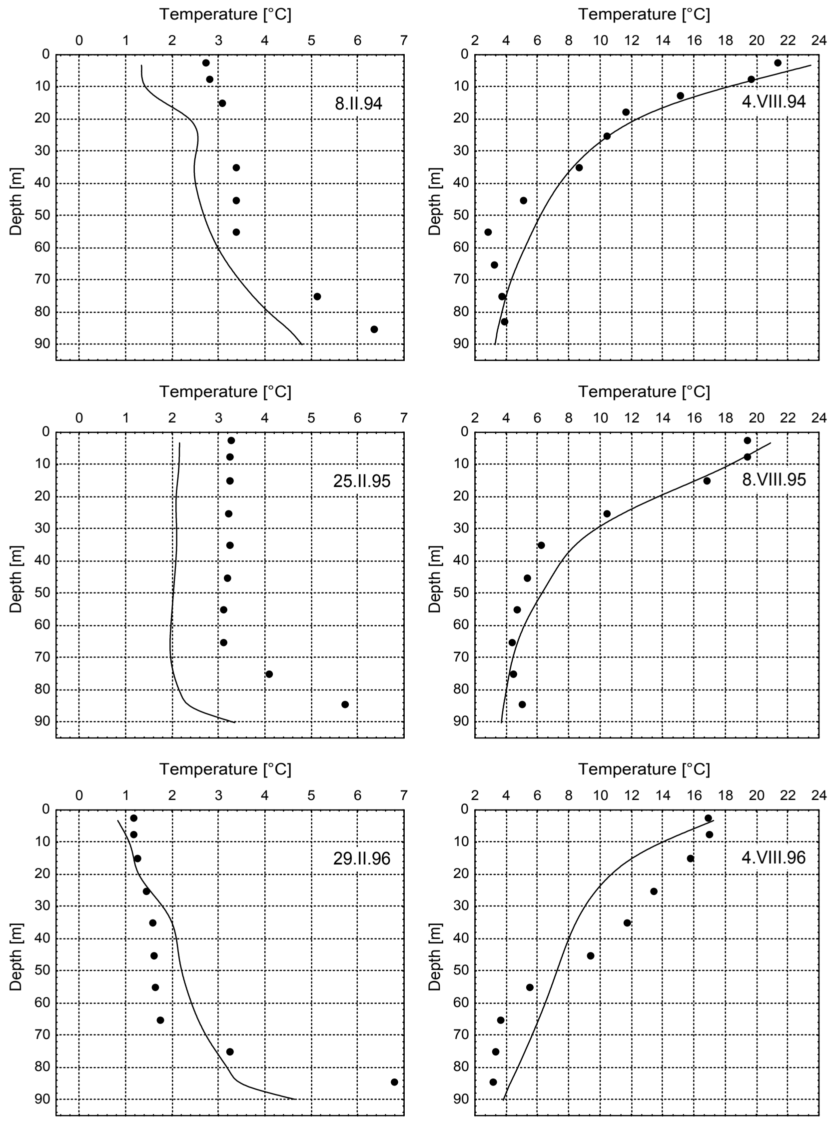


Fig. 5. The vertical distributions of calculated (line) and observed (points) temperature at station P116 in winter and summer 1994–1996

layer, a good conformity of calculated and measured temperature and salinity values has been achieved. The differences increase gradually with depth. The greatest errors are found below 60 m (in halocline layer and below it) where the salinity values are markedly underestimated. In the 2nd and 3rd year of simulation, a tendency to gradual underestimation of calculated salinity occurs for the all layers. These errors occur probably due to underestimation of water

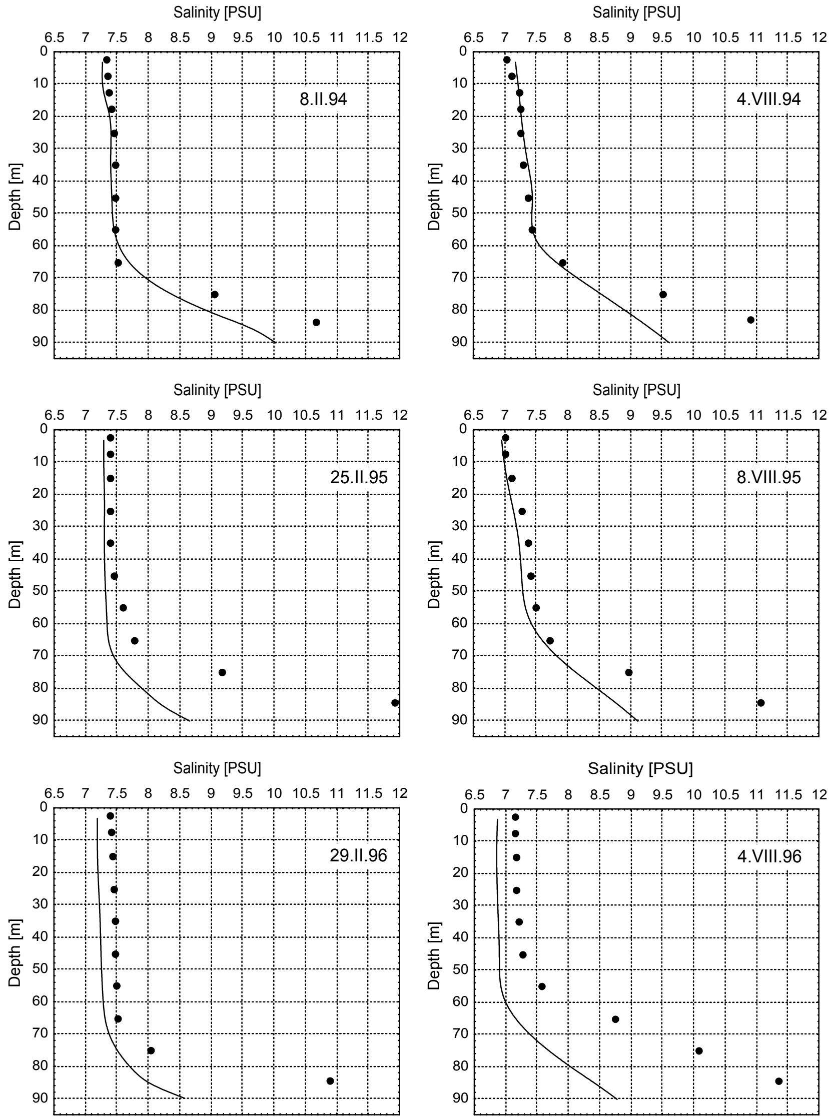


Fig. 6. The vertical distributions of calculated (line) and observed (points) salinity values at station P116 in winter and summer 1994–1996

exchange with the North Sea caused by simplification of boundary conditions at the open Baltic boundary. The model correctly approximates the seasonal changes of depth of thermo- and halocline occurrence. The highest conformity of calculated and measured values are noted in the western and southern parts of the Gulf, the lowest – near the open boundary between the Gulf of Gdańsk and the Baltic Sea.

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